The PTF Modeling Framework

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Insureware

Innovative Statistical Solutions for P&C Insurance

Contents

Introduction

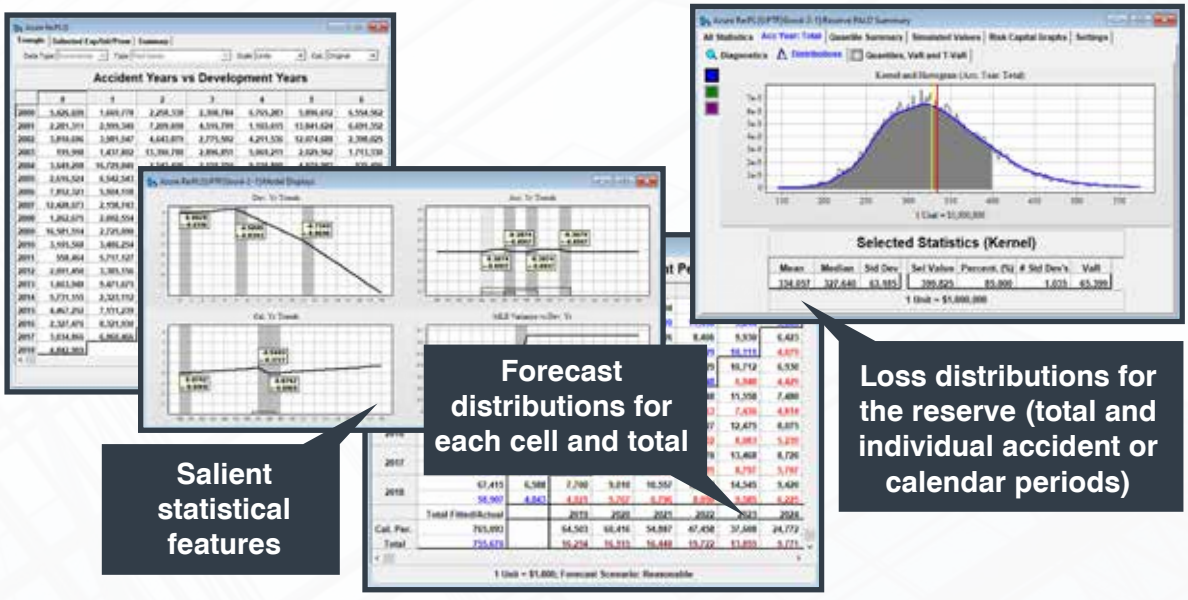
In the Probabilistic Trend Family (PTF) modeling framework, we identify a parsimonious model describing the trends in the three directions (development, accident, and calendar), along with the volatility about the trend structure.

The identified model in the PTF modeling framework:

- **measures calendar period trends (sum of economic and social inflation);**
- **separates trends from volatility and quantifies both;**
- **fits a probability distribution to every cell;**
- **forecasts distributions for every cell going forward;**
- **provides calendar year liability stream and its distributions, and other metrics;**
- **is NOT a method.**

Unlike other actuarial solutions, a PTF model is not pre-defined but rather is tailored to each company's data. This enables the tool to extract the salient statistical features of any loss development array and produce meaningful results for all aspects of the liability distributions. Any modeling assumptions can be tested and verified that they are supported by the data.

The PTF modeling framework gives insight into your business, extracting knowledge to make informed decisions. Foresight does not depend on lucky charms.



Maximize the value of your data by engaging Insureware's software solutions. Know when to reduce or increase exposure (or premium), purchase reinsurance, or enter (exit) a market.

Importance of Calendar trends

Insurance claims arrive in calendar time. This fundamental feature means that any model for insurance data without the capacity to adjust for calendar trends cannot provide a sound modeling framework.

The Probabilistic Trend Family (PTF) decomposes loss development arrays into:

- **trends in the three directions**
  - **development,**
  - **accident, and**
  - **calendar,**
- **and the volatility about the trend structure.**

Further, insurance losses are subject to inflationary trends. Inflationary trends are, by definition, multiplicative in calendar time.

Traditional methods, based on link ratios, actually assume an average, constant, calendar period trend. However, this trend is never quantified.

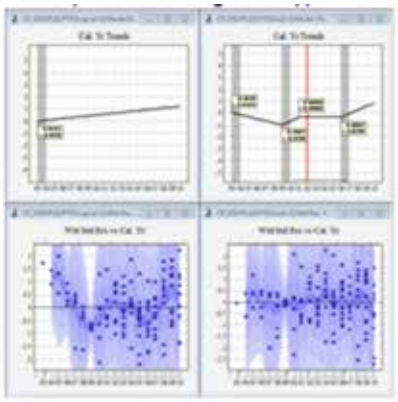
This leaves the user(s) of these methods exposed to unknown trends with no knowledge to be able to account for changes by calendar year.

Adjusting for economic inflation does not fix this as social inflation, or superimposed inflation, is typically a stronger effect than economic inflation.

Another important consideration is that inflation does not change the percentage volatility. That is, on the log-scale, inflation is additive.

We contrast the difference between quantifying the actual calendar year trends and assuming an average calendar year trend in a real-life Commercial Property case study (company identifiability removed).

Left: an average calendar year trend is fitted to all years **Right**: calendar year trend changes are applied as they occur



The residuals (difference between the trends in the data and the trends in the method) illustrated under the model trend displays, shows the average trend (left) does not describe the ups and downs in the calendar year trends. In contrast, the model on the right captures these changes - the residuals are centered around the zero line. The second model provides a much better description of the trends in the data.

Note the difference in the final calendar year trends!

The model on the left applies a 6.9%+_1.3% calendar trend per calendar year whereas the model on the right measures a 30%+_3.2% calendar year since 2006. The effect of these two contrasting calendar year trend assumptions on the future liability stream, and the total reserve distribution, is significant.

Data, distributions and diagnostics

Any incremental loss development array (or triangle) can be modeled in the PTF modeling framework:

- **Paid Losses (PL),**
- **Allocated Loss Adjustment Expenses (ALAE),**
- **Incurred Losses (IL),**
- **Case Reserve Estimates (CRE),**
- **Number Cases Reported (NCR), and**
- **… many more!**

Each data type can be utilized to construct a story about the data – providing critical understanding to the company.

Lognormal distributions are fitted to each cell in the triangle. Trends link the means in the cells. The volatility characteristics of the data are reflected in the standard deviations in each cell. Together, these measures provide the foundation of the exposure to risk (reserves, risk capital, liability stream, and so forth) in the triangle.

Models are not fitted without considering whether they are appropriate for the data. If you have seen a PTF model, you have seen **one** PTF model. There are an infinite set of models that can be built in the framework, but only one that describes the features going on in your business is an appropriate one for your company.

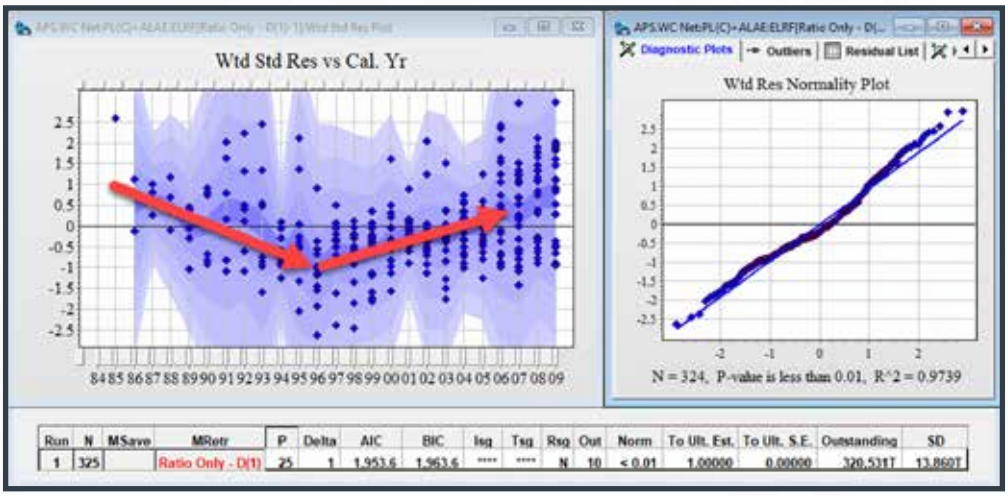
The model fits well if the data can be regarded as a sample from the fitted model. Intuitive diagnostics enable the actuarial analyst to ensure the fitted PTF model is suitable.

These diagnostics include:

- **Residuals by development period, accident period, and calendar period.**
- **Normality plots and histograms,**
- **Autovalidation tools for testing trend stability,**
- **Forecast tables with historical fitted values vs observed values along with the projection of future losses, and**
- **Graphs of observed historical trends vs future trend assumptions.**

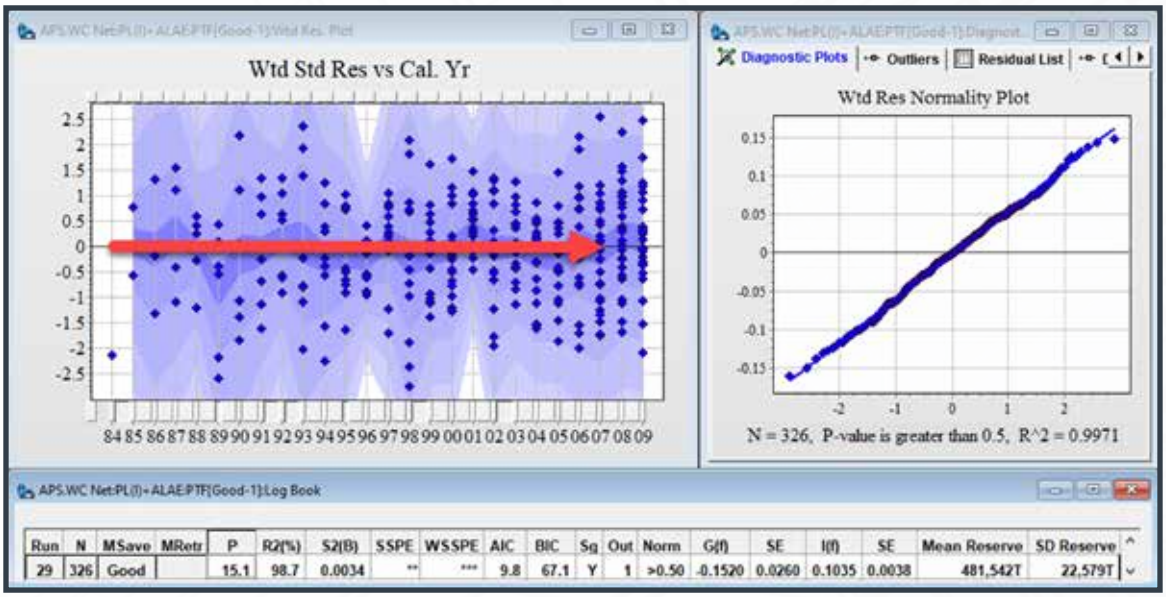
All of these make for a powerful argument to support the selections the actuarial analyst has made – for the identified model in the modeling framework, and for any future trend assumptions.

In the Extended Link Ratio Family (ELRF) modeling framework, the residuals versus calendar year show a downward trend then an increasing trend. The residuals also exhibit poor normality (right hand side).



This company, if using link ratio methods, would be suffering adverse development every year. The calendar year trends are not described by the link ratio method and they are stuck in constant ‘catch up’ mode. This is in fact exactly what happened in this scenario. Insureware was consulted to evaluate this company’s loss portfolio and quickly showed what was really happening…

Contrast this with the equivalent diagnostics after an optimal model has been identified in the PTF modeling framework. The residuals are centred on zero (by calendar period), and the test for normality is achieved.



The difference in total mean reserve is substantial – 481M using a forecast scenario continuing with the trends identified in the Probabilistic Trend Family modeling framework vs 320.5M using when using the Mack method in the Extended Link Ratio Family. The missing factor in the ELRF model were the changes in social inflation – the most recent calendar year trend measured in the PTF modeling framework is 10% per year! This is critical information for the company. If realized, the company would have exited this WC portfolio, or substantially raised premiums) soon after the social inflation began in earnest (2002). Millions of dollars could have been saved.

Do link ratio methods work for your data? A small investment into the ELRF™ software suite may surprise you!

The many benefits and advantages of PTF:

- **Decomposition of trends into:**
    - o development period trends,
    - o accident period trends, and
    - o calendar period trends.
- **Volatility measurements by development period and accident period**
- **Extensive diagnostics to validate models and provide confidence in actuarial projections**
- **Distributions by cell, totals (accident and calendar), plus grand total**
- **Liability stream by calendar year (IFRS 17 and any investment program)**
- **Trends can be extended outside the triangle period**
    - o Use the same model for reserving and pricing!
    - o Get 'what if' analyses and inflation impact assessments in real time
- **Simulate from the forecast to complete:**
    - o Risk capital analysis,
    - o Asset liability matching,
    - o Solvency II risk capital,
    - o Reinsurance assessment, and
    - o … much more!

With detailed trend and volatility information, actuarial teams can provide valuable insight into the insurance business.

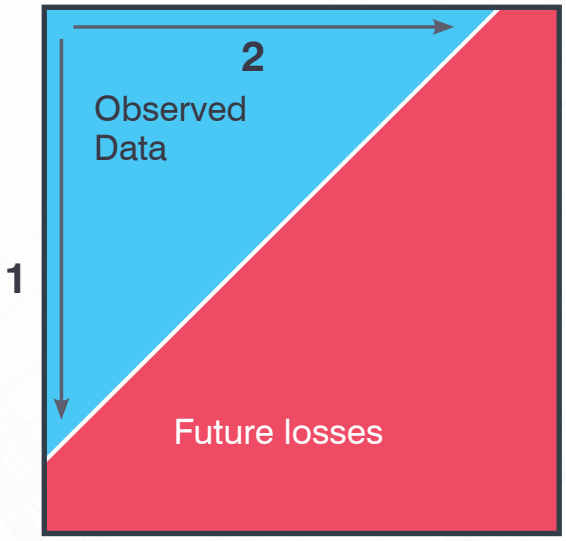
The remainder of this brochure is more technical in nature and discusses:

- **the loss development array problem and why the PTF modeling framework is a natural solution;**
- **how to build a PTF model while mitigating model specification risk;**
- **forecast scenario selection and evaluating sensitivity to calendar year trends; and**
- **the one-year reserve risk and other results.**

The loss development array problem

Fundamentally, all we are interested in is the total amount to be paid in each underwriting (or accident period) period and the level of risk associated with each estimate. The total mean forms our best estimate of how much capital the company needs to pay for losses (on average), and the risk capital measures the additional capital required for the company to cover losses above the mean to a particular risk comfort level.

Since there is return on investment on the held capital, the total payments expected in each year are necessary for optimal asset liability matching and similarly required by IFRS 17 for appropriate profit/loss recording. This necessitates that we examine the predictions for incremental losses and ensure they make sense.



The totals have a distribution of possible outcomes with all the features of a distribution: mean, standard deviation, and other properties. In order for the distributions to be meaningful, the calculations must be directly related to the individual cells, and the projection of the individual cells must be relatable to every other cell in the triangle. This is what the PTF modeling framework does.

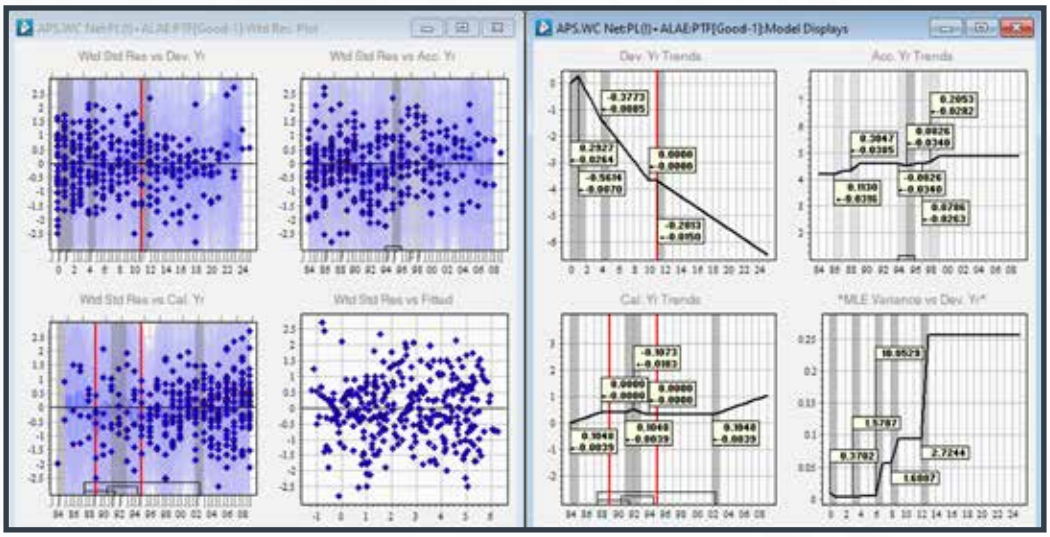
Calendar period trends project across the accident and development periods. This feature of economic and social inflation is axiomatic and any modeling tool applied to actuarial loss triangles must be able to measure the sum of the inflation so that future forecasts can be assessed for reasonableness.

The PTF modeling framework lets you measure the inflation actually in the data. This inflation may match prior expectations, but in our experience, social inflation rarely manifests exactly how our intuition might lead us. The value of the knowledge of what is really going on cannot be underestimated. The company mentioned previously underpriced a portfolio by over 100M.

Building a PTF model

Identifying a model in the PTF modeling framework is a straightforward exercise. A **modeling wizard** accelerates the process ensuring actuarial results can be delivered in a timely fashion. An interactive interface enables quick and easy customization of the model. Assumptions for the future are explicit, relatable to past experience, auditable, and able to be monitored on updating.

The displays in PTF are interactive. Changes in trends can be added, or tested, by simply clicking on the display. Similarly, points can be removed from the model estimation process with a simple click.



The residual display provides instant visual indication of whether there are any missing trends in the data. If the model is describing the data, the scatter in the plots will be random without pattern. In the example illustrating diagnostics, the residuals seen in the ELRF modeling framework still had trends (the residuals were increasing). Above, and in the model identified previously for the other company, the residuals are randomly scattered around the trends. The models are describing what is going on.

The model display on the right shows the trends and changes identified in the data. Knowing the trends in each direction, and what is driving the changes in trend (for instance, the sharp drop in calendar year 2007), enables the actuary to make any changes to the forecast to reflect future expectations.

All the trends and volatility measures identified in the model, can be customized for the future. Both the mean estimates and standard deviations of future trends can be modified along with volatility changes by accident period or development period.

Actuarial judgement is enhanced by providing sound statistical support for selections!

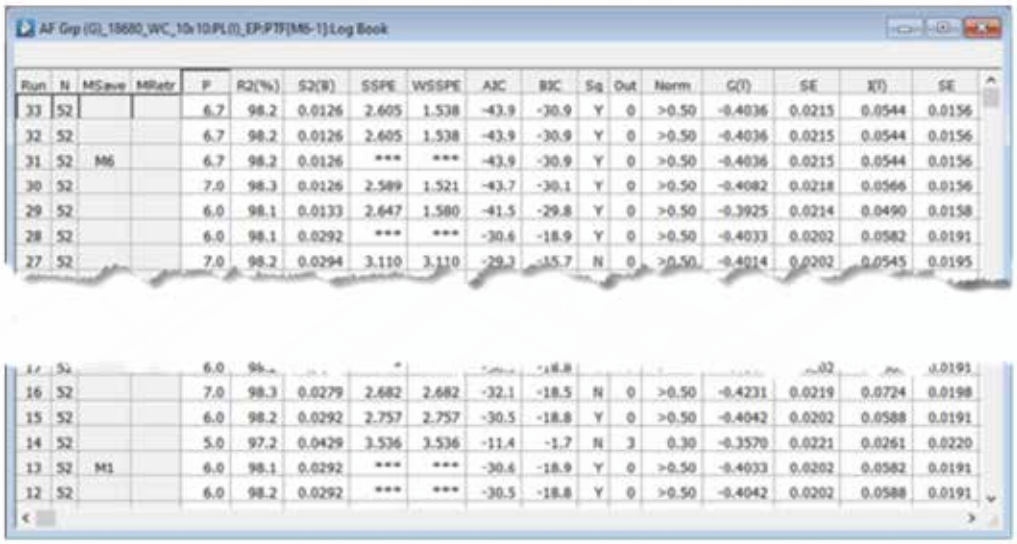
A modeling wizard

A modeling wizard is available for identifying models in the Probabilistic Trend Family modeling framework. The modeling wizard considers all aspects of modeling within the AI script:

- Addition of trend parameters
- Adjustment for volatility changes in the development direction
- Outliers

Many different models are considered in the process of producing three final models for the analyst to evaluate. The models the wizard saves into the database, especially the final model, have the lowest Bayes Information Criterion (BIC) of the models the wizard generates.

For instance, consider the logbook extract below. The modeling wizard has estimated around 30 models in the process of determining the final 'best' model found - M6. The model has the best BIC of -30.9.



Run	N	MSave	MRetr	P	R2(%)	S2(B)	SSPE	WSSPE	AIC	BIC	Sg	Out	Norm	G(I)	SE	X(I)	SE
33	52			6.7	98.2	0.0126	2.605	1.538	-43.9	-30.9	Y	0	>0.50	-0.4036	0.0215	0.0544	0.0156
32	52			6.7	98.2	0.0126	2.605	1.538	-43.9	-30.9	Y	0	>0.50	-0.4036	0.0215	0.0544	0.0156
31	52	M6		6.7	98.2	0.0126	***	***	-43.9	-30.9	Y	0	>0.50	-0.4036	0.0215	0.0544	0.0156
30	52			7.0	98.3	0.0126	2.589	1.521	-43.7	-30.1	Y	0	>0.50	-0.4082	0.0218	0.0566	0.0156
29	52			6.0	98.1	0.0133	2.647	1.580	-41.5	-29.8	Y	0	>0.50	-0.3925	0.0214	0.0490	0.0158
28	52			6.0	98.1	0.0292	***	***	-30.6	-18.9	Y	0	>0.50	-0.4033	0.0202	0.0582	0.0191
27	52			7.0	98.2	0.0294	3.110	3.110	-29.3	-15.7	N	0	>0.50	-0.4014	0.0202	0.0545	0.0195
16	52			7.0	98.3	0.0279	2.682	2.682	-32.1	-18.5	N	0	>0.50	-0.4231	0.0219	0.0724	0.0198
15	52			6.0	98.2	0.0292	2.757	2.757	-30.5	-18.8	Y	0	>0.50	-0.4042	0.0202	0.0588	0.0191
14	52			5.0	97.2	0.0429	3.536	3.536	-11.4	-1.7	N	3	0.30	-0.3570	0.0221	0.0261	0.0220
13	52	M1		6.0	98.1	0.0292	***	***	-30.6	-18.9	Y	0	>0.50	-0.4033	0.0202	0.0582	0.0191
12	52			6.0	98.2	0.0292	***	***	-30.5	-18.8	Y	0	>0.50	-0.4042	0.0202	0.0588	0.0191

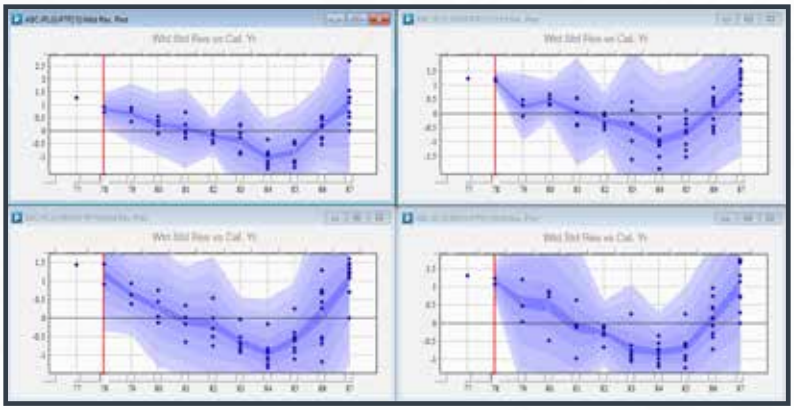
The modeling wizard is scripted in such a way as to emulate the model building process of a human analyst.

This tool should be seen as an accelerator and not a replacement for model design. The wizard models provide a great first model to explain the salient features of any given triangle. However, the final consideration of the model, whether the trends and volatility changes, identified accurately reflects the features in the data remains the responsibility of the analyst.

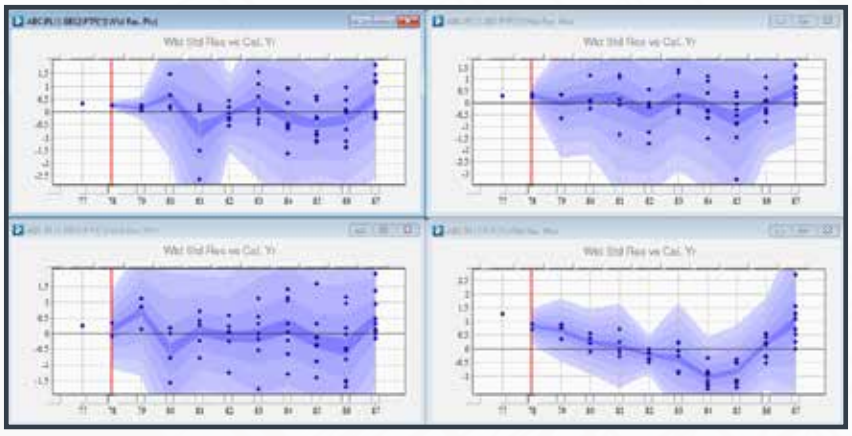
Mitigating model specification risk

The Probabilistic Trend Family model identification process mitigates model specification risk by design. If the residuals are not randomly distributed about the trends, then there are features of the data the model is not describing. Any mismatch between the model features and the loss data are quickly identifiable during peer review and oversight.

A good model describes the data so, in one respect, the data can be regarded as a one-path simulation from the model. This means that simulations from the model should be indistinguishable from the real data with respect to their critical statistical features. All of the datasets below exhibit the same calendar structure. The identified PTF model described the changing calendar year trends.



Simulation from the Mack methods, say using the bootstrap, produces samples that are very different from the real data when there are changes in inflation. The bootstrap samples from the Mack method do not have the same calendar year trends as the real data.



Cumulation masks the calendar period trend emergence (information is lost), and distributional assumptions made by the regression methods are almost never held by the data. The real data is not a likely path from the model. Using link ratio methods, including Mack, result in high model specification risk.

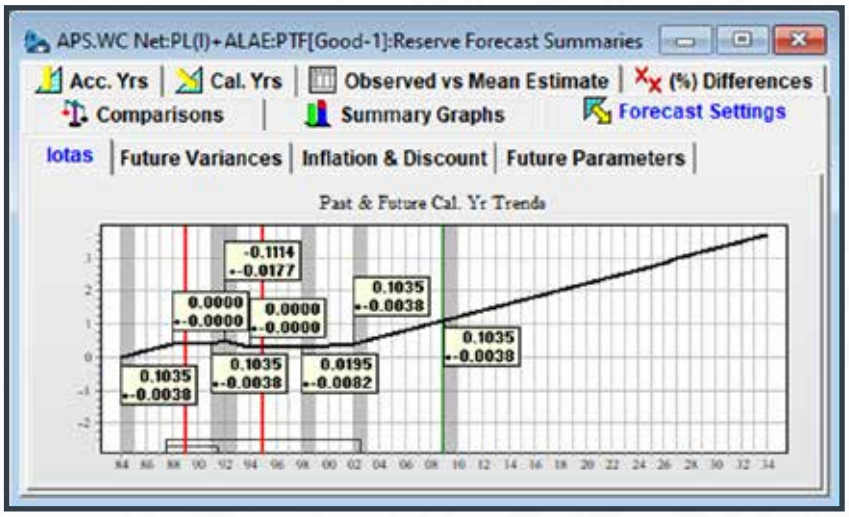
Designing a future forecast scenario

All aspects of the future forecast are customisable by the actuary.

These include:

- **Future calendar period trends;**
- **Future development trends;**
- **Future accident period levels; and**
- **Future volatility measures.**

The parameter with the most impact is typically the future calendar year parameter. To get the best estimate, it is most important to have the most reasonable distribution for this parameter going forward. Below, the future calendar year is set to continue at 10.35%+_0.38% for the remainder of the run-off period.



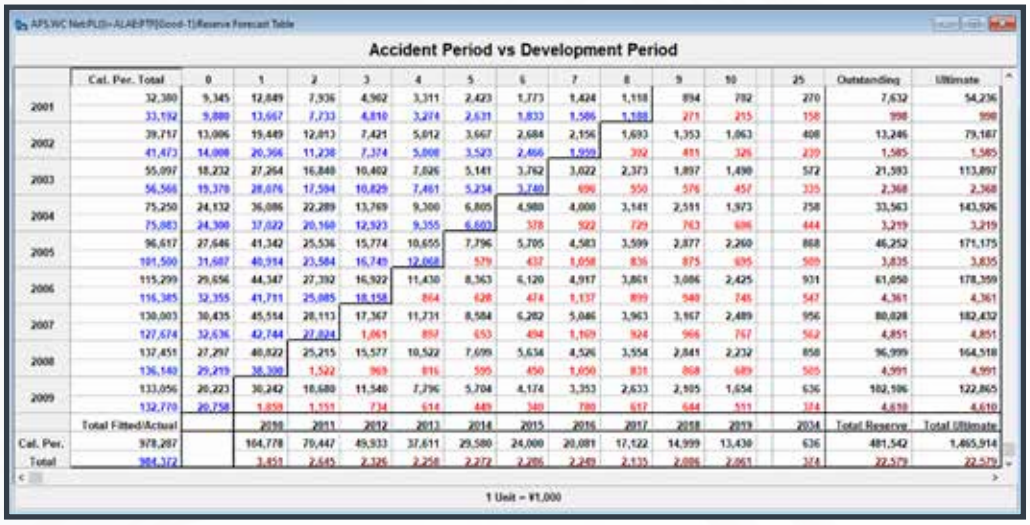
Forecast scenarios can be compared for sensitivity – eg what is the impact of a 2% increase (or decrease) in future calendar year trend? These scenarios can be run on the fly and/or automatically via the COM API.

If the future calendar year trend decreases by 2%, the total reserve mean is reduced from 481M to 429M. However, if the calendar year trend increases by 2%, then the total reserve mean is increased from 481M to 548M. Because of the long-tail nature of this portfolio, the future calendar year trend has significant impact. Note to reach the Mack method's mean estimate of 320M, a trend of 2.5%+_ would need to be assumed. The likelihood of the 10.35%+_ trend, which has been stable for seven years, suddenly dropping to 2.5% for the remainder of the run-off period is very unlikely.

The sensitivity of forecast scenarios can be compared to the one year reserve risk.

Forecast tables and loss distributions

One of the critical tables in ICRFS™ is the forecast table. This table is provided in all the modeling frameworks in ICRFS™. This table, and the accident year summary table containing the one-year reserve risk, are two important results providing critical information to the actuary when deriving the best estimate of the total reserve distribution.



Accident Period vs Development Period

	Cal. Per. Total	0	1	2	3	4	5	6	7	8	9	10	25	Outstanding	Ultimate
2001	32,380	9,345	12,849	7,936	4,962	3,311	2,423	1,773	1,424	1,118	894	782	270	7,632	54,236
	33,192	9,889	13,667	7,733	4,810	3,274	2,631	1,833	1,586	1,188	271	215	158	998	998
2002	39,717	13,006	19,449	12,013	7,421	5,012	3,667	2,684	2,156	1,693	1,353	1,063	408	13,246	79,187
	41,473	14,008	20,366	11,238	7,374	5,008	3,523	2,466	1,959	392	411	326	239	1,585	1,585
2003	55,097	18,232	27,264	16,848	10,402	7,026	5,141	3,762	3,022	2,373	1,897	1,490	572	21,593	113,897
	56,566	19,379	28,076	17,594	10,829	7,461	5,234	3,740	696	550	576	457	335	2,368	2,368
2004	75,250	24,132	36,086	22,289	13,769	9,300	6,805	4,980	4,000	3,141	2,511	1,973	758	33,563	143,926
	75,883	24,300	37,022	20,160	12,923	9,355	6,603	578	502	729	763	606	444	3,219	3,219
2005	96,617	27,646	41,342	25,536	15,774	10,655	7,796	5,705	4,583	3,599	2,877	2,260	868	46,252	171,175
	101,500	31,687	40,914	23,584	16,749	12,068	579	437	1,058	836	875	695	509	3,835	3,835
2006	115,299	29,656	44,347	27,392	16,922	11,430	8,363	6,120	4,917	3,861	3,086	2,425	931	61,050	178,359
	116,385	32,355	41,711	25,085	18,158	864	628	474	1,137	899	940	745	547	4,361	4,361
2007	130,003	30,435	45,514	28,113	17,367	11,731	8,584	6,282	5,046	3,963	3,167	2,489	956	80,028	182,432
	127,674	32,636	42,744	27,024	1,061	897	653	494	1,169	924	966	767	562	4,851	4,851
2008	137,451	27,297	40,822	25,215	15,577	10,522	7,699	5,634	4,526	3,554	2,841	2,232	858	96,999	164,518
	136,140	29,219	38,300	1,522	969	815	595	450	1,050	831	868	689	505	4,991	4,991
2009	133,056	20,223	30,242	18,680	11,540	7,396	5,704	4,174	3,353	2,633	2,105	1,654	636	102,596	122,865
	132,770	20,758	1,859	1,151	734	614	449	340	780	617	644	511	374	4,610	4,610
	Total Fitted/Actual		2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2034	Total Reserve	Total Ultimate
Cal. Per.	978,287		164,778	70,447	49,933	37,611	29,580	24,000	20,081	17,122	14,999	13,430	636	481,542	1,465,914
Total	984,372		3,451	2,645	2,326	2,258	2,272	2,206	2,249	2,135	2,006	2,061	374	22,579	22,579

1 Unit = ¥1,000

Interpreting the table:

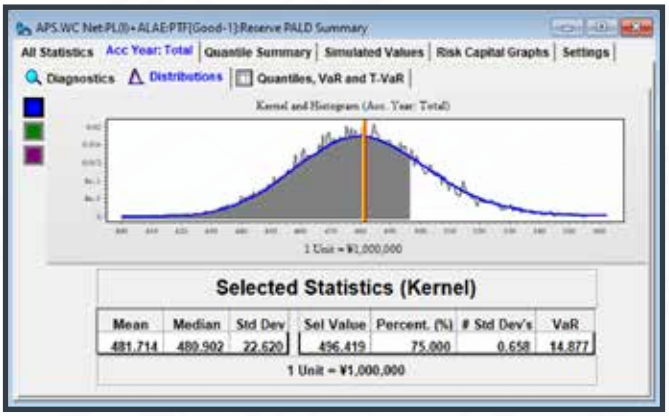
- **Blue numbers are the observed values (historical losses in this example),**
- **Black numbers are the fitted/projected means in each cell (fitted in the historical portion; projected in the future),**
- **Red / Burgundy numbers are standard deviations in each cell, or for the total of cells by accident period, calendar period or total period. These values are not standard deviations of the mean.**
- **Calendar year totals are shown down the bottom of the table whereas accident year totals are shown on the right.**

The future liability stream (calendar year totals) can be compared to the calendar years losses the company has just paid. Here the loss of 105M in 2010 is perhaps lower than expected than the 133M paid in 2009 even when 20M paid in development year zero is included. However, as the company had begun reducing exposure to this portfolio, the reduction in the total losses was deemed reasonable.

2007	130,003	30,435	45,514	28,113	16,056	10,027	6,783	4,589	3,408	2,474	1,828	1,328	157	53,586	155,990
	127,674	32,636	42,744	27,024	980	767	516	361	789	577	558	409	92	2,550	2,550
2008	137,451	27,297	40,822	23,312	13,314	8,315	5,625	3,806	2,826	2,052	1,516	1,101	130	67,748	135,267
	136,140	29,219	38,309	1,407	829	645	435	304	656	480	463	349	77	2,836	2,836
2009	133,656	20,223	27,959	15,967	9,119	5,695	3,852	2,607	1,936	1,405	1,039	754	89	74,363	95,121
	132,770	20,758	1,719	583	580	449	303	212	450	329	318	233	53	2,946	2,946
	Total Fitted/Actual		2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2024	Total Reserve	Total Ultimate
Cal. Per.	978,287		96,868	60,213	39,458	27,477	19,978	14,986	11,593	9,138	7,401	6,126	89	323,168	1,307,540
Total	984,372		3,191	2,261	1,838	1,650	1,534	1,428	1,298	1,139	1,029	949	53	11,389	11,389

Contrast the liability stream to match 323M (roughly the Mack method mean) with the liability stream from the PTF modeling framework. The losses in the next calendar year having a mean of 96M is extremely improbable. This table is a powerful diagnostic to assess the reasonableness of any forecast.

Further, as log-normal distributions are projected for every future cell, simulations can be run to estimate the total reserve distribution. The probability of loss given particular levels of reserves are held, along with appropriate thresholds of risk capital can be expressly calculated.

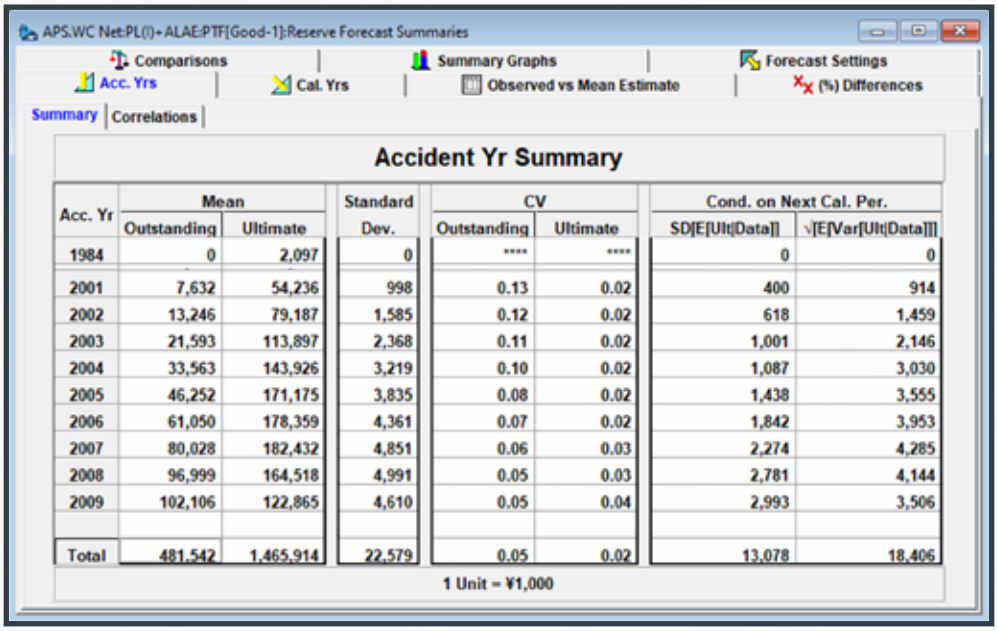


A complete picture of the total reserve distribution and/or distributions by individual accident periods or calendar periods can be readily obtained.

Pricing future underwriting years uses the same model as the reserving team and can incorporate risk margins into the total premium charged based on the level of the risk arising from the particular line.

One year reserve risk

An important question for any actuary is ‘how much variation can I expect in the reserve estimates when next year’s data come in’? Insurance data are volatile and this leads to variation in the mean ultimates as each diagonal is added. If the losses come from the predicted distributions, then the expected variation in mean ultimate can be calculated.



Accident Yr Summary

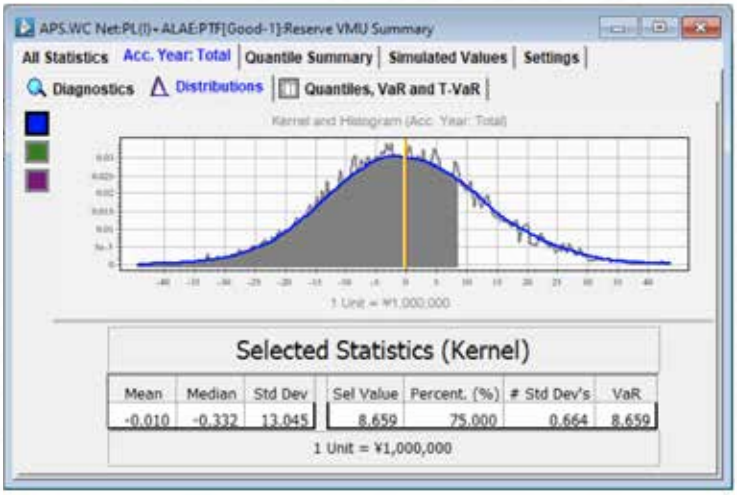
Acc. Yr	Mean		Standard	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate	Dev.	Outstanding	Ultimate	SD[E[Ult|Data]]	√[E[Var[Ult|Data]]]
1984	0	2,097	0	****	****	0	0
2001	7,632	54,236	998	0.13	0.02	400	914
2002	13,246	79,187	1,585	0.12	0.02	618	1,459
2003	21,593	113,897	2,368	0.11	0.02	1,001	2,146
2004	33,563	143,926	3,219	0.10	0.02	1,087	3,030
2005	46,252	171,175	3,835	0.08	0.02	1,438	3,555
2006	61,050	178,359	4,361	0.07	0.02	1,842	3,953
2007	80,028	182,432	4,851	0.06	0.03	2,274	4,285
2008	96,999	164,518	4,991	0.05	0.03	2,781	4,144
2009	102,106	122,865	4,610	0.05	0.04	2,993	3,506
Total	481,542	1,465,914	22,579	0.05	0.02	13,078	18,406

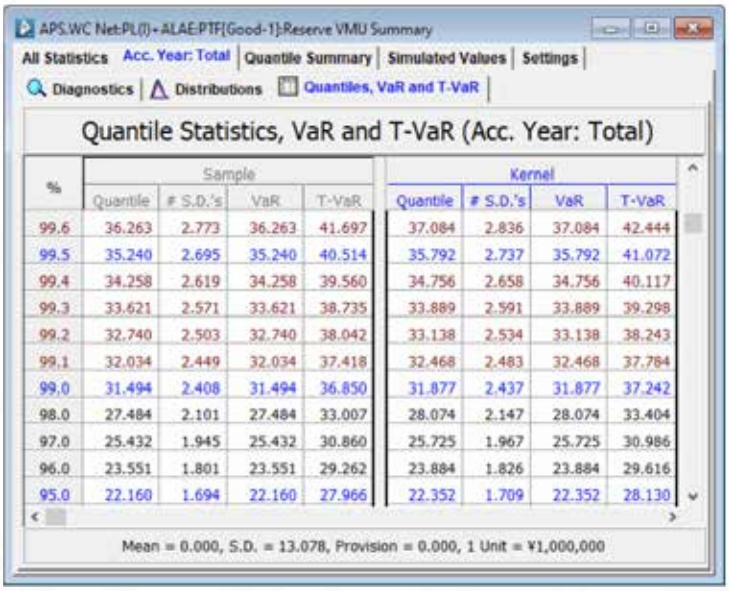
1 Unit = ¥1,000

Accident (or underwriting) year summaries in the PTF modeling framework, as illustrated above, include two columns that are conditional on the next calendar period’s data. The first of these columns, SD[E[Ult|Data]], is a measure of the variation in mean ultimate after observing the next diagonal – assuming that the losses arrive according to the forecast scenario trends (and volatility). The second column shows the remaining variation in the forecast after the next calendar period’s data are observed. Note we are not limited to conditioning on one period, but can condition on multiple periods.

Distribution of the Variation in Mean Ultimate

The distribution of the variation in mean ultimate can be calculated by simulating the next years losses and reapplying the model and forecast scenarios. As with the Predictive Aggregate Lognormal Distributions (PALD) module, the Value-At-Risk, Tail-Value-At-Risk and other metrics can be computed.

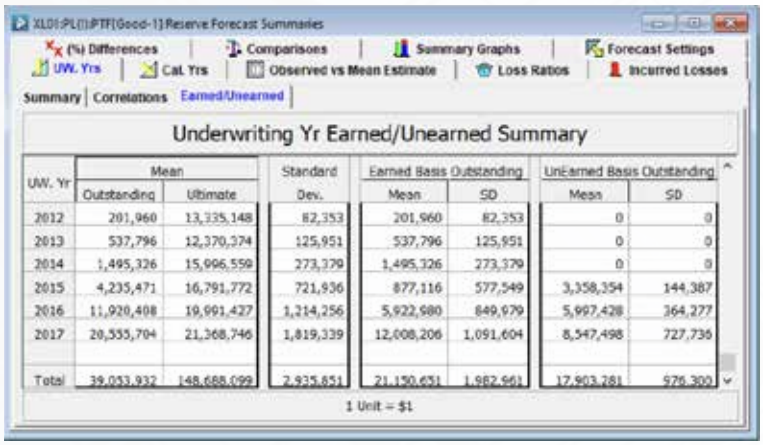


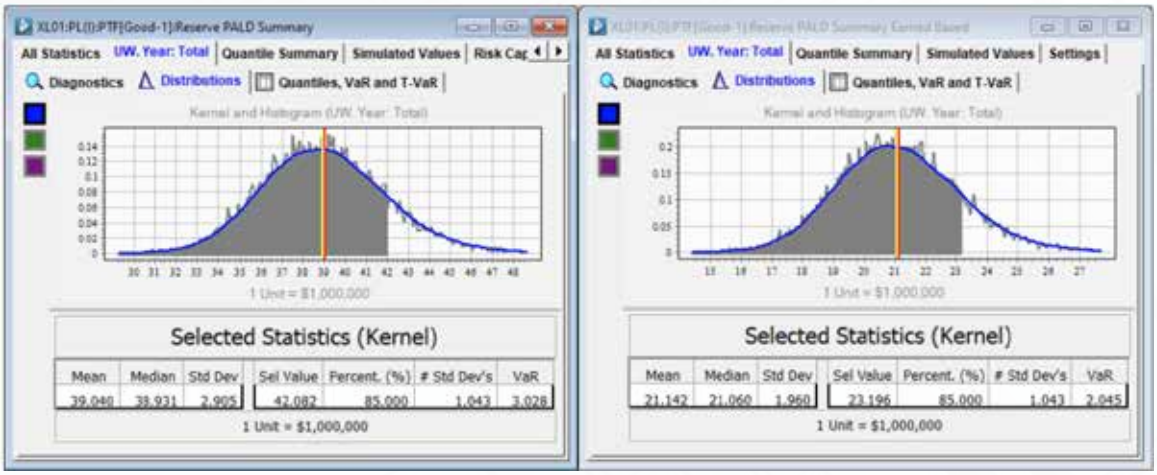


The 99.5th percentile of the variation in mean ultimate can be used as part of the standard formula for Solvency II purposes.

Earned vs Unearned risk

Projected loss distributions can be seperated into earned and unearned risk.





Transforming analytics

Insureware's software platform, ICRFS™, is a complete solution for all aspects of long-tail liability risk management. From aggregation of data into triangles, analysis of trends, volatility, and correlations, to generation of results into reports, ICRFS™ has it all.

Some people have the idea that companies need to use complicated and convoluted software to warehouse data, to analyze it, and to modernize actuarial systems. This software invariably needs to be customised to the company's individual requirements as no 'off-the-shelf' package would be sufficient to meet their needs. After years of implementation costs, staff changes, and many hundreds of thousands of dollars later… it turns out that they should have simply licenced ICRFS™!

The PTF modeling framework allows companies to interrogate their loss development arrays in a way that maximises the extraction of information about a line of business's trends and volatility - especially the level of social inflation. This information needs to be transferred into knowledge and wisdom so informed decisions can be made regarding reserves, risk, pricing, and reinsurance solutions.

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About Insureware

Insureware is not your typical long-tail liability risk management firm: we are R&D focused. Our team of world-class statisticians originated many of the ideas that the industry now aspires to. They have published numerous papers not only in actuarial journals but also in preeminent statistical journals. Insureware creates and supports the only comprehensive, enterprise wide, long-tail liability risk management software in the world.

Insureware has advised on a wide-range of insurance matters including:

- Reserve due diligence;
- Mergers and Acquisitions;
- Assessing risk capital and Solvency II capital requirements for submissions to regulators and rating agencies;
- Underwriting and pricing; and
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www.insureware.com

Email: info@insureware.com
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